Theory of

Electric and Magnetic Field Transducers

Frits Buesink
University of Twente,
EEMCS-HSC[1]
Education Legacy

[1] Electrical Engineering, Mathematics and Computer Science (EEMCS)
Historic Study Collection (HSC)
See https://studieverzameling.utwente.nl/

1

Electric field transducer theory

1.1 Gauss' law

Electric fields can be measured using a "capacitive plate" antenna. This is essentially a sheet (or any other shape) of metal with dimensions much smaller[1] than a fraction of the shortest wavelength of the fields to be measured. When exposed to an electric field, any conductive object will develop a surface charge. If not connected to anything this is accomplished by moving free electrons to or from the exposed surface depending on the polarity of the field. For a static electric field, the process is known as the basis of charging by induction, the source of many electrostatic discharge (ESD) problems. When the exposed plate is connected to e.g. a grounded wire, a current will flow to bring in or remove the charge required to re-establish the equilibrium. The magnitude of this charge, Q_D, is given by Equation 1.1.

[1] Would it be larger, resonance effects can be expected to change the behavior described in this text. The relatively simple approach used here is called a "low-frequency approximation".

If the electric field is alternating, this induced charge will follow the instantaneous field and change accordingly. This changing charge manifests as an alternating *displacement*-current[2], $I_{DISPLACE}$, as expressed in Equation 1.2. This equation is derived using Gauss' law and illustrated in Fig. 1.1 which actually uses Gauss' law in the opposite direction. The theorem is normally described as follows:

"The net electric flux, Ψ, through any hypothetical closed surface is equal to $\frac{1}{\varepsilon_0}$ times the net electric charge, Q_D within that closed surface."

[1, p. 514]

An additional simplification is the assumption that the electric field lines are all in parallel and perpendicular to the proposed sensor plate and *only* affect the left side of the sensor in Fig. 1.1. The enhancement with a second ground reference plane (GRP) plane in Fig. 1.2 is an attempt to at least shield electric fields on the other side of the sensor-plate.

In the frequency domain, the displacement current is evaluated as Equation 1.3:
$\varepsilon_0 = \frac{1}{36\pi} \cdot 10^{-9} = 8.84 \cdot 10^{-12}$ F/m is the permittivity of vacuum [3].

[2] Displacement as there is no galvanic connection. The product of the electric field and the permittivity of the environment is called the "dielectric displacement": $D_{incident}(t) = E_{incident} \cdot \varepsilon_0$

[3] Would the sensing plate be immersed in some liquid i.e. another medium than air or vacuum, this permittivity must be multiplied with the relative permittivity of this medium: ε_r.

$$Q_D = A_E \cdot \varepsilon_0 \cdot E_{incident}(t) \tag{1.1}$$

$$\begin{aligned} I_{DISPLACE}(t) &= \frac{\mathrm{d}Q_D(t)}{\mathrm{d}t} = \frac{\mathrm{d}(A_E \cdot \varepsilon_0 \cdot E(t))}{\mathrm{d}t} = \\ &= A_E \cdot \varepsilon_0 \cdot \frac{\mathrm{d}E(t)}{\mathrm{d}t} \end{aligned} \tag{1.2}$$

$$\begin{aligned} I_{DISPLACE}(j\omega) &= j\omega \cdot Q_D(j\omega) = \\ &= j\omega \cdot A_E \cdot \varepsilon_0 \cdot E(j\omega) \end{aligned} \tag{1.3}$$

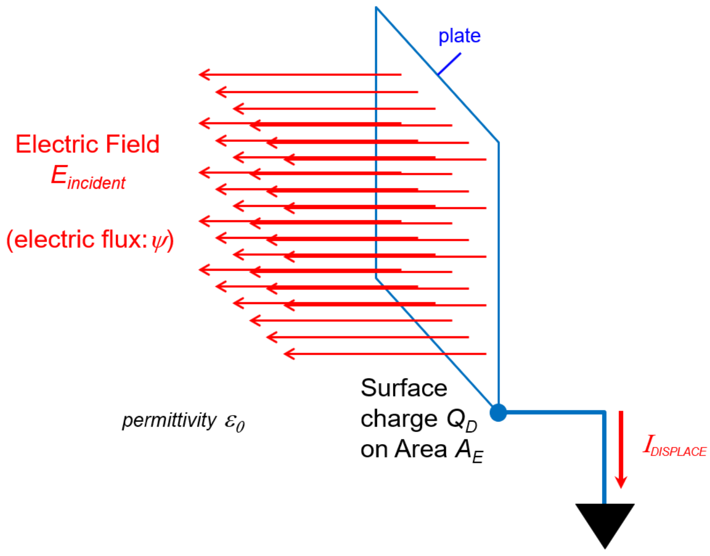


Fig. 1.1: Current induction into metal plate exposed to electric field

1.2 The proposed electric field transducer

A single metallic rod or plate can be used as an electric field probe. This type of probe reacts to electric fields from all directions. To focus the measurements in a specific direction (shield fields from other directions) the probe should be enhanced with a second plate as shown in Fig. 1.2. This second plate is usually connected to the return conductor/shield of the transducer-cable and actually acts as a ground reference plane (GRP).

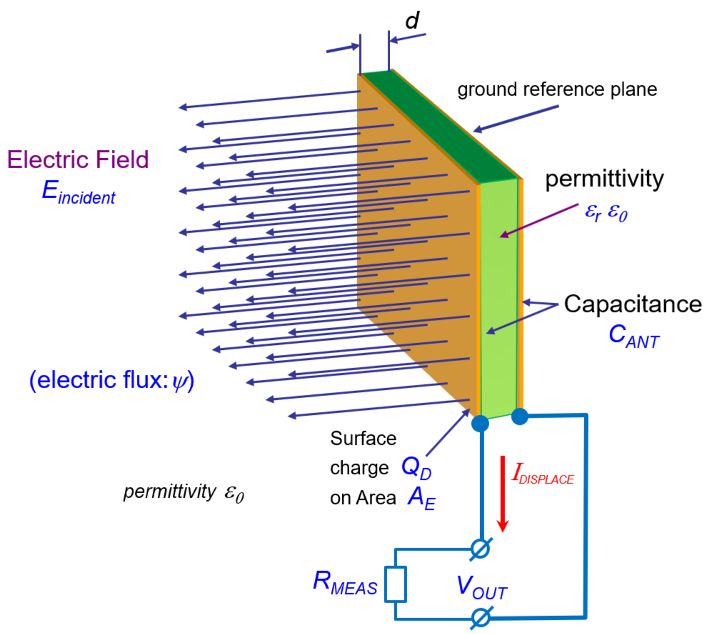


Fig. 1.2: Capacitive plate antenna

The sensor plate now is part of a capacitor formed by both plates sandwiching a dielectric separator with a relative permittivity of ε_r. An electrical circuit model can now be made of the capacitive antenna as shown in Fig. 1.3.

The equivalent electrical circuit of the transducer in Fig. 1.3 contains a, so-called, substitute current source, $I_{DISPLACE}$, in addition to the passive elements C_{ANT} and R_{MEAS}. The substitute current source represents the effect of Gauss' law. The sensitivity of this electric probe/transducer

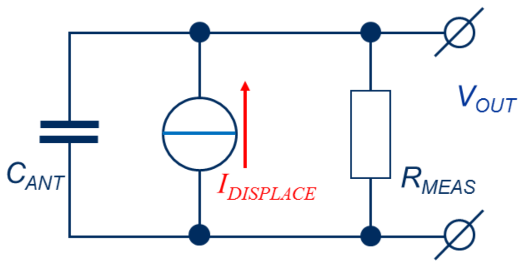


Fig. 1.3: Equivalent circuit model for the capacitive plate antenna

is usually expressed as an antenna factor (AF), the ratio of the electric field, $E_{incident}$ and the resulting transducer output voltage, V_{OUT}. This AF is found from Fig. 1.3 using Kirchhoff's electrical voltage and current laws [1, p. 599] and Equation 1.3 as Equation 1.4.

$$
\begin{aligned}
ANT = \frac{E_{incident}(j\omega)}{V_{OUT}(j\omega)} &= \\
&= \frac{\varepsilon_r \cdot [j\omega R_{MEAS}C_{ANT} + 1]}{d \cdot j\omega R_{MEAS}C_{ANT}} \quad \text{m}^{-1}
\end{aligned}
\tag{1.4}
$$

For frequencies lower than the corner frequency $\omega = (R_{MEAS}C_{ANT})^{-1}$, labeled LF, Equation 1.4 turns into Equation 1.5. For frequencies above that corner frequency, labeled HF, Equation 1.6 is found.

$$
AF_{ANT}(\text{LF}) = \frac{\varepsilon_r}{d \cdot j\omega R_{MEAS}C_{ANT}} \quad \text{m}^{-1} \tag{1.5}
$$

$$
AF_{ANT}(\text{HF}) = \frac{\varepsilon_r}{d} \quad \text{m}^{-1} \tag{1.6}
$$

The behavior of the transducer's AF as function of frequency looks as the graph shown in Fig. 1.4.

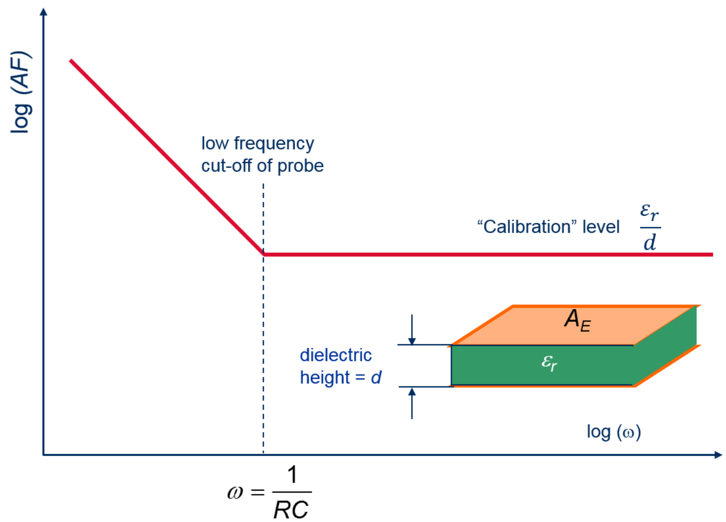


Fig. 1.4: The antenna factor of an electric field probe

1.3 Example *home-made* electric field antenna

An antenna was built using standard FR4 epoxy-glass printed circuit board (PCB) material with a dielectric thickness of $d = 1.6$ mm. The area of the plate is $A_E = h \cdot w = 0.227 \times 0.307 = 0.07$ m^2 as can be seen from Fig. 1.5 and 1.6. The finished product is shown in Fig. 1.7. The measured capacitance is $C_{ANT} = 2$ nF which, given $\varepsilon_0 = 8.84 \cdot 10^{-12}$ allows the calculation of ε_r using Equation 1.7.

$$C_{ANT} = \frac{A_E \cdot \varepsilon_r \cdot \varepsilon_0}{d} = 2 \cdot 10^{-9} \quad \text{F} \tag{1.7}$$

$$\varepsilon_r = 5.2 \quad - \tag{1.8}$$

Given the dielectric thickness $D = 1.6$ mm and the relative permittivity $\varepsilon_r = 5.2$, the flat part of the antenna factor becomes $AF_{ANT}(HF) = 3250$ m^{-1}. Note that this will be the high-frequency value of any capacitive transducer built from this PCB base material. The area A_E in combination with the terminating resistor R_{MEAS} only determine the actual cut-off frequency.

Fig. 1.5: The physical antenna with height measure

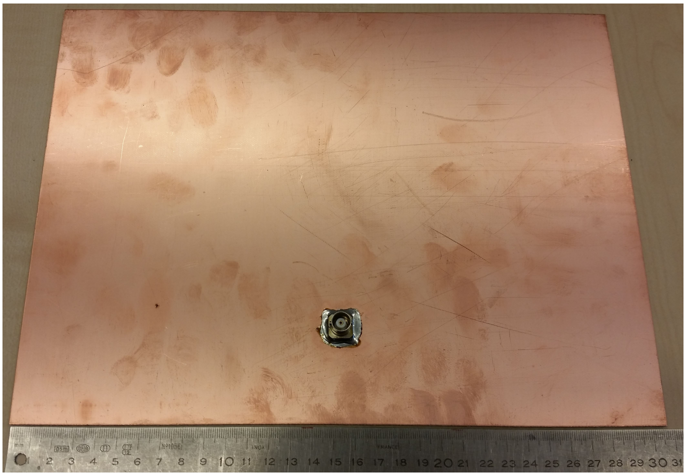


Fig. 1.6: The physical antenna with width measure

Fig. 1.7: The finished antenna with PVC protection

Substituting Equation 1.7 into Equation 1.5 provides Equation 1.9.

$$AF_{ANT}(\text{LF}) = \frac{1}{j\omega R_{MEAS} \cdot A_e \cdot \varepsilon_0} \quad \text{m}^{-1} \tag{1.9}$$

The cross-over frequency, F_{CO}, shown in Equation 1.10 marks the low end of the very low-frequency (3 to 30 kHz) (VLF) frequecy region where the antenna has the constant $AF_{ANT}(HF) = 3250$ m^{-1} specified by Equation 1.6. This frequency is a *designable parameter* and should be selected. Below that value the sensitivity of the antenna decreases proportional with frequency (i.e. the antenna factor (AF) goes up).

F_{CO} is used to calculate R_{MEAS} in Equation 1.11, given the measured antenna capacitance of Equation 1.7.

$$F_{CO} = (2\pi R_{MEAS} C_{ANT})^{-1} = 2000 \quad \text{Hz} \tag{1.10}$$

with Equation 1.7

$$\text{this leads to } R_{MEAS} \approx 39 \quad \text{k}\Omega \tag{1.11}$$

The antenna factor of the realized transducer can now be calculated from Equations 1.5, 1.6 and 1.7. It is shown in the graph of Fig. 1.8. The essential parameters for this transducer are summarized below:

1. Base material standard FR4 epoxy-glass PCB material with a dielectric thickness of $d = 1.6$ mm;
2. Relative permittivity of base material: $\varepsilon_r = 5.2$
3. Area: $A_E = 69.7 \cdot 10^{-3}$ m^2
4. Internal capacitance $C_{ANT} = 2$ nF
5. Antenna factor(flat part) $AF_{ANT}(HF) = 3250$ m^{-1}
6. Cut-off frequency $F_{CO|}{}_{(R_{MEAS}=39\text{k}\Omega)} = 2$ KHz

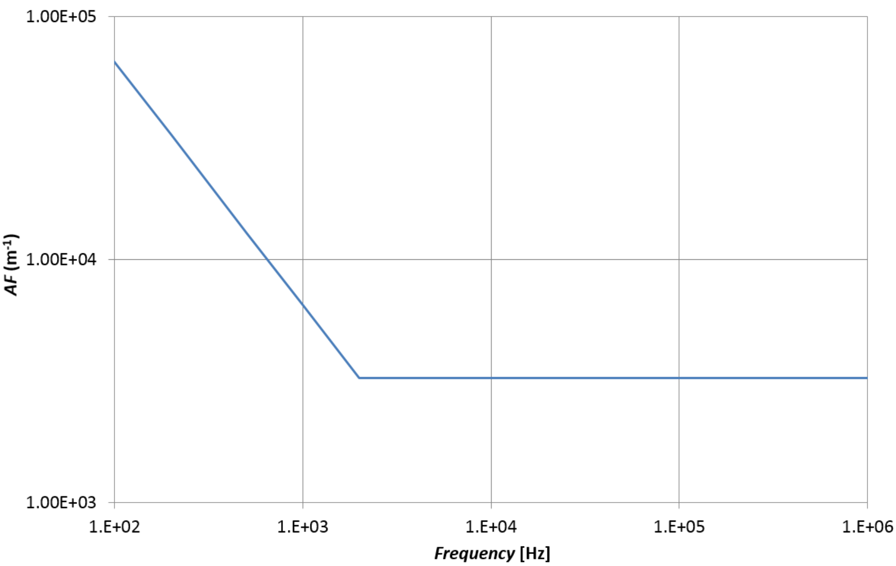


Fig. 1.8: Antenna Factor against frequency of the large plate-antenna

1.4 Simulating the behavior of an E-field tranducer

Using the free simulator LTSPICE from Analog Devices the behavior of e.g. the capacitive probe of Fig. 1.7 can be simulated. The simulator can be downloaded at: https://www.analog.com/en/design-center/design-tools-and-calculators/ltspice-simulator.html#.

To model the transducer a replacement needs to be designed for the coupling into the electric field that can be implemented with the available voltage and current sources in LTSPICE.

Based on the antenna factor $AF_{ANT}(HF) = 3250$ m^{-1} a unity electric field strength of $E_{incident} = 1$ V/m will produce an output voltage in the flat part of the transfer function of $V_{OUT} = \frac{1}{AF_{ANT}(HF)} = 308$ μV or -70 dBV. The coupling can be modeled as a small "environment" capacitor, C_{ENV} connected to a *voltage* source, $V_{ENV} = 1$ V while driving the antenna capacitance, C_{ANT} at its other terminal. This actually is a capacitive voltage divider. The value of this small capacitor can be estimated with the following assumptions:

- In front of the antenna assume a plate with the same dimensions as the plate antenna under study;
- The hypothetical plate is in parallel to the transducer plate at a distance of $L = 1$ m and their centers are aligned;
- Electric field lines are straight lines perpendicular to the two plates and present only between the plates (no fringing effects)[4];
- The capacitance, C_{ENV} between the plates can now be calculated using the parallel plate capacitance Equation 1.12;

$$C_{ENV} = \frac{A_E \cdot \varepsilon_0}{L} = 0.62 \text{ pF} \tag{1.12}$$

- The hypothetical environment plate is connected to a voltage source of $V_{ENV} = 1$ V.

[4] The model tries to mimic an infinite environment with an electric field level of 1 V/m which is not disturbed by the presence of the transducer.

The LTSPICE model of the large capacitive transducer in Fig. 1.7 is shown in Fig. 1.9.

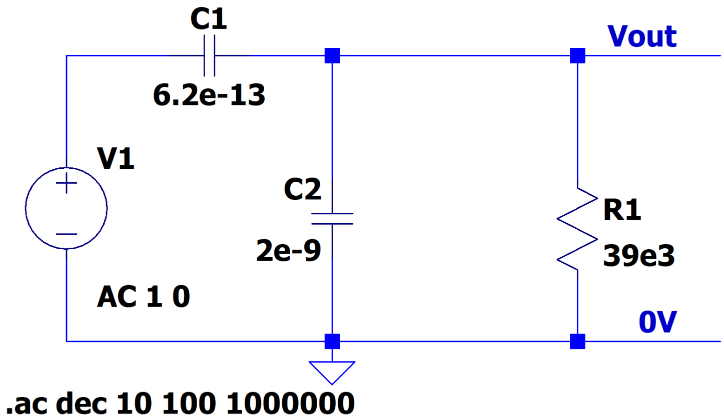


Fig. 1.9: LTSPICE model for the large capacitive plate transducer

The model has a source voltage level of $V1 = 1$ V hence the expected asymptotical (high-frequency) output voltage is $V_{OUT} = 308$ μV $= -70$ dB/V as shown in Fig. 1.10. Keep in mind that any passive transducer will show amplitude *and* phase responses.

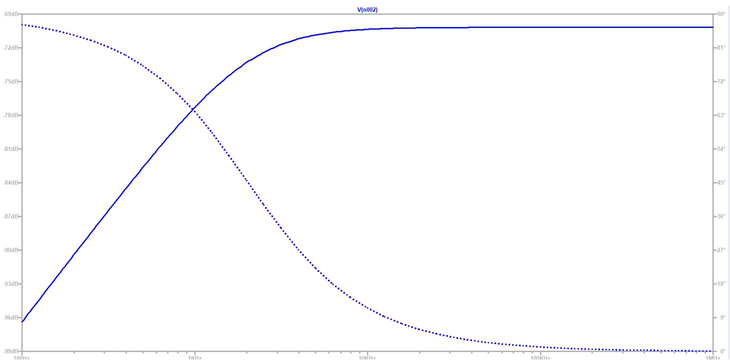


Fig. 1.10: Simulated large capacitive plate transducer response

1.5 A smaller electric field “sniffer-probe”

A smaller model of an E-field probe is shown in Fig. 1.11. Fig. 1.12 shows the insulated version which is a good idea if measurements near live electronics are planned.

Fig. 1.11: Smaller Electric field pick-up transducer

Fig. 1.12: Fully insulated Electric field pick-up transducer

The parameters for this transducer are:

1. Base material standard FR4 epoxy-glass PCB material with a dielectric thickness of $d = 1.6$ mm;
2. Relative permittivity of base material: $\varepsilon_r = 5.2$
3. Diameter: $D = 60$ mm;
4. Area: $A_E = 2.8 \cdot 10^{-3}$ m^2

5. Internal capacitance $C_{ANT} = 72$ pF

6. Antenna factor(flat part) $AF_{ANT}(HF) = 3250$ m^{-1}

7. Cut-off frequency $F_{CO}|_{(R_{MEAS}=50\Omega)} = 278$ MHz

1.6 Tuning the usable frequency range of capacitive probes

As demonstrated in Equation 1.10 and 1.11, the load or termination resistor can be tuned to obtain a specific desired cut-off frequency, F_{CO}. Another option could be to increase the parallel capacitance, C_{ANT}, by making it larger, by reducing the dielectric thickness, d, in Figure 1.2 or using a higher permittivity material to increase ε_r. But it is even possible to use an external capacitor in parallel to the termination resistor, R_{MEAS}. This is an option e.g. in situations where this resistor cannot be modified. In this case it would be better to write Equation 1.5 as Equation 1.13 and use Fig. 1.2 in combination with Fig. 1.13.

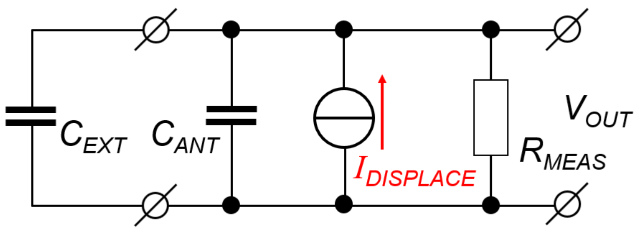


Fig. 1.13: Equivalent circuit model with external capacitor

$$
\begin{aligned}
AF_{ANT} =& \frac{E_{incident}(j\omega)}{V_{OUT}(j\omega)} = \frac{j\omega(C_{EXT}+C_{ANT})R_{MEAS}+1}{j\omega A_E \varepsilon_0 R_{MEAS}} = \\
=& \frac{j\omega(C_{EXT}+\frac{A_E\varepsilon_r\varepsilon_0}{d})R_{MEAS}+1}{j\omega A_E\varepsilon_0 R_{MEAS}} \quad \text{m}^{-1}
\end{aligned}
\tag{1.13}
$$

The cut-off frequency in this situation is given in Equation 1.14.

$$F_{CO_{w-ext-cap}} = \frac{1}{2\pi \left[C_{EXT} + \dfrac{A_E \varepsilon_r \varepsilon_0}{d} \right] R_{MEAS}} \qquad \text{Hz} \tag{1.14}$$

The high-frequency asymptote of the antenna factor can now be written as Equation 1.15.

$$AF_{ANT}(HF) = \frac{C_{EXT}}{A_E \varepsilon_0} + \frac{\varepsilon_r}{d} \qquad \text{m}^{-1} \tag{1.15}$$

Equation 1.15 indicates that the antenna factor is increased with a positive amount $\frac{C_{EXT}}{A_E \varepsilon_0}$ i.e. the sensor voltage output will be decreased with respect to the version without external capacitor.

1.7 On the label “D-dot probe”

The measuring principle of these electric field probes is based on the derivative of the “Dielectric displacement” with time $\dot{D}(t) = \frac{\mathrm{d}D(t)}{\mathrm{d}t} = \frac{\mathrm{d}(\varepsilon_0 \cdot E(t))}{\mathrm{d}t}$ and for that reason are sometimes referred to as “Ddot” probes, e.g. in MIL-STD-461 [2, page 152].

2

Current and magnetic field transducer theory

2.1 Mechanism of mutual induction

Currents can be measured using a current-clamp, a current-probe, or, more general, a current-transducer.

For direct current (DC) measurements these instruments are often performed using a semiconductor element based on the *Hall-effect* (see e.g. https://en.wikipedia.org/wiki/Hall_effect). To measure only alternating current (AC), the instruments are often based on *mutual-induction*.

Mutual induction is the mechanism that transformers are based on: two (or more) coils are placed close together such that the magnetic flux (Φ) of one of the coils will also penetrate the core of the other coil(s). For two adjacent loops, this is shown in Fig. 2.1.

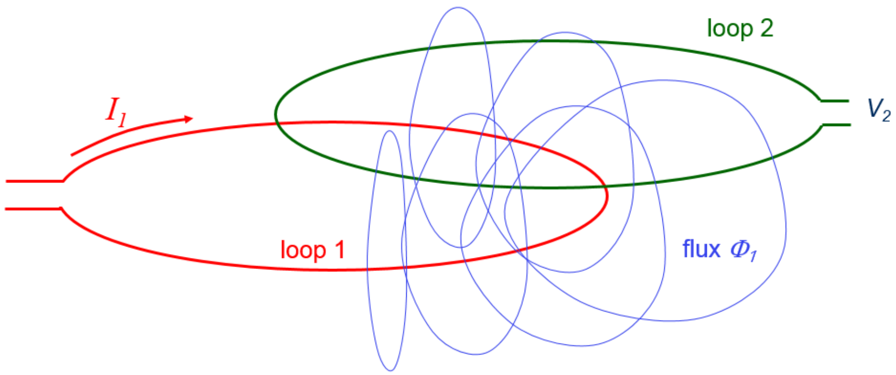


Fig. 2.1: The principle of mutual induction

The current I_1 in loop 1 in Fig. 2.1 generates an amount of magnetic flux. The ratio of this flux and the initiating current is called the (self) induction L_1 of loop (or coil) 1 as shown in Equation 2.1.

$$L_1 = \frac{\Phi_1}{I_1} \qquad \text{H (Henry)} \tag{2.1}$$

Let us assume an amount Φ_2 of the flux produced by loop 1, brought about by I_1, is "enclosed" by loop 2. Then the mutual inductance of loop 1 and loop 2 is M_{12} as shown by Equation 2.2.

$$M_{12} = \frac{\Phi_2}{I_1} \qquad \text{H} \tag{2.2}$$

To build a transformer the two coils are usually wound on a common (closed loop) magnetizable core to maximize the amount of flux that eventually reaches loop (coil) 2. The common symbol for a transformer is shown in Fig. 2.2 (possible magnetizable core not shown).

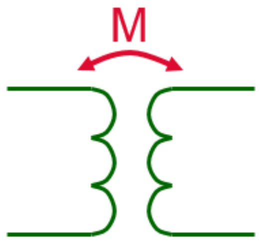


Fig. 2.2: Flux coupled coils form a transformer

To measure a current in a wire, this wire is actually led through the closed core on which the measuring coil is wound. This is shown in Fig. 2.3.

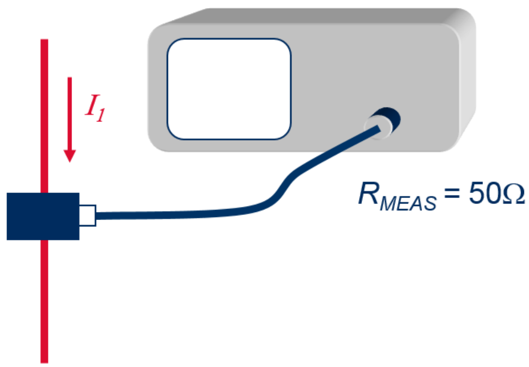


Fig. 2.3: Coil-on-closed-core to measure current in wire

2.2 Model for a current transducer using a substitute source

As shown in Fig. 2.1, the measuring coil (loop 2 in the figure) produces a voltage V_2 for a given current I_1. To evaluate this voltage, Equation 2.2 can be rewritten as Equation 2.3. It is assumed here that M_{12} is not dependent on time (t).

$$M_{12} \cdot I_1(t) = \Phi(t) \qquad \text{Wb (Weber)} \tag{2.3}$$

To find the voltage produced by the measuring instrument coil, Faraday's Law [1, p. 659] as shown in Equation 2.4 for the time-domain is used.

$$V_2(t) = -\frac{\mathrm{d}\Phi(t)}{\mathrm{d}t} = -\frac{\mathrm{d}\left[M_{12} \cdot I_1(t)\right]}{\mathrm{d}t} \qquad \text{V} \tag{2.4}$$

Alternatively, this Equation 2.4 can be written in the frequency domain as Equation 2.5 which is more appropriate if the behavior in the frequency domain needs to be evaluated. As above for the time domain, the value of M_{12} is assumed fixed i.e. independent of frequency (this should be checked at high frequencies especially if a magnetizable core is used). This is done to keep the equations simple.

$$V_2(j\omega) = -j\omega\Phi(\omega) = -j\omega M_{12} \cdot I_1(j\omega) \qquad \text{V} \tag{2.5}$$

Equation 2.5 is a so-called substitute voltage source, describing the behavior of the measuring coil based on its mutual induction with the, usually one-turn, "primary" coil of the current-transformer. In the usable pass-band of the transducer the mutual induction $M_{12}(\omega)$ is supposed to be a constant and is simply written as M_{12}.

The substitute source replaces the operation of Faraday's law in loop 2 in Fig. 2.2 and is depicted in Fig. 2.4.

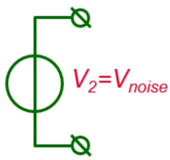


Fig. 2.4: Substitute source replacing the effect of Faraday's law

2.3 Frequency response of a current transducer

To determine what the output of the current-probe or current-transformer will be the complete circuit diagram of the setup is needed. This diagram is shown in Fig. 2.5. On the left-hand side, the substitute voltage source of Fig. 2.4 is drawn.

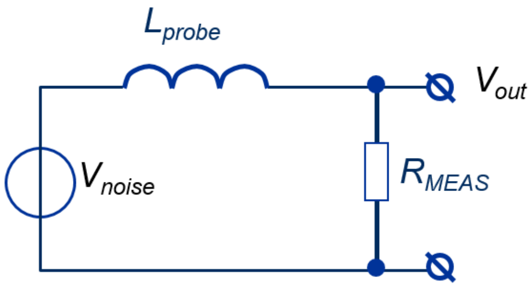


Fig. 2.5: Equivalent diagram of the current measurement setup

The schematic diagram also shows the inductance of the measuring instrument, L_{probe} and a terminating resistor, R_{MEAS}. This resistor is normally placed at the end of the cable connecting the current transducer to the oscilloscope or spectrum analyzer that is used to visualize the result as shown in Fig. 2.3.

The output voltage, V_{out}, can now be evaluated as usual with the help of both the Kirchhoff voltage and current rules [1, p. 599]. Dividing this voltage by the current, I_1, required to generate it provides us with the *transfer-impedance* of the current transducer shown in Equation 2.6.

$$Z_T(j\omega) = \frac{V_{out}(j\omega)}{I_1(j\omega)} = j\omega M_{12} \cdot \frac{\dfrac{R_{MEAS}}{L_{probe}}}{j\omega + \dfrac{R_{MEAS}}{L_{probe}}} \quad \Omega \qquad (2.6)$$

To facilitate drawing a graph of this the result Equation 2.6 can be split into two equations providing asymptotes for low and high frequency behavior. Key is the probe cut-off angular frequency ω_{CO} shown in Equation 2.7 or frequency in Equation 2.8.

$$\omega_{CO} = \frac{R_{MEAS}}{L_{probe}} \qquad {}^{rad}\!/_{s} \tag{2.7}$$

$$F_{CO} = 2\pi \frac{R_{MEAS}}{L_{probe}} \qquad \text{Hz} \tag{2.8}$$

For low-frequencies, Equation 2.6 changes into Equation 2.9. For high frequencies, Equation 2.10 emerges.

$$Z_T(LF) = j(2\pi f) \cdot M_{12} \qquad \Omega \qquad f << F_{CO} \tag{2.9}$$

$$Z_T(HF) = \frac{M_{12} R_{MEAS}}{L_{probe}} \qquad \Omega \qquad f >> F_{CO} \tag{2.10}$$

Graphically, these results are shown in Fig. 2.6.

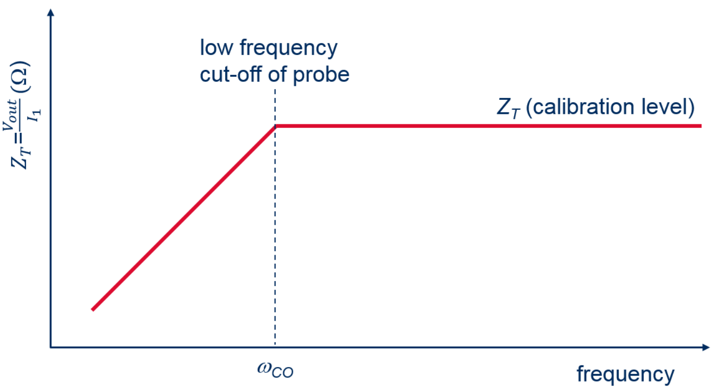


Fig. 2.6: Asymptotical current transducer response

M_{12} is supposed to be a constant with respect to frequency. This is indicated by the flat horizontal part of Fig. 2.6. At very high frequencies this is no longer true. This implies that once the transducer is built the high-end of the flat part of the transfer-impedance has to be determined experimentally (or: “calibrated”).

2.4 Example *home-made* current transducer

A current transducer with $F_{CO} \approx 300$ Hz was built using a toroid ferrite core 3E27 size TX40/24/16 (Farnell #1784186). Wound with 53 turns of insulated American wire gauge (AWG) 22 wire (diameter $d = 1.3$ mm including insulation). The measured inductance is $L_{PROBE} = 27.2$ mH which will result in a cut-off frequency of $f_{CO} = 293$ Hz using a $50\ \Omega$ terminating impedance.. The finished transducer is shown in Figure 2.7.

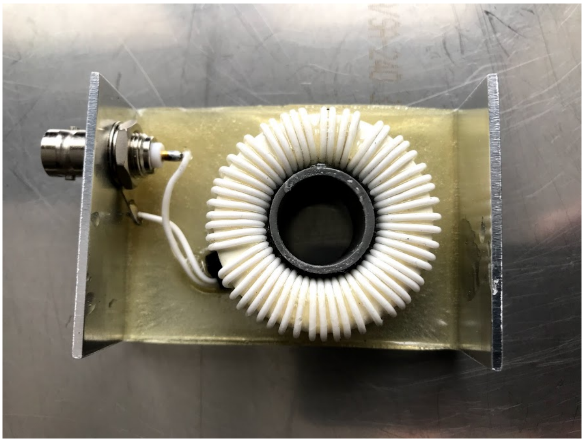


Fig. 2.7: Finished transducer 53 turns AWG22 on TX40/24 3E27 core

The plastic center tube could be left out because it is important that possibly used cable connectors in experiments fit through the inner hole. Without inner tube the center hole is 20 mm. The plastic[1] tube makes the manufacturing process easier: a strip of sticky tape is placed over the hole in the aluminum frame with the BNC connector and the bottom of the plastic tube is pressed against it to stop the glue from running out. The standard 20 mm installation pipe has enough room for BNC connectors. Sticky tape is also used to cover the sides of the frame. Araldite glue is then used to cover the core and fix it to the frame. It is advised to cure the Araldite in an oven at $T_{OVEN} \approx 60°$C To use wider connectors the plastic tube could be drilled out. Or a larger core could be used. Farnell has a wider model available with the same core

[1] May be metal. Avoid full conductive shorts around the core.

material: the TX80/40/15-3E27 with order code #1784197. This has a 40 mm inner hole which has ample room for the standard IEC mains connector (even after winding) and allows measurements of Common-Mode (CM)-mains currents.

The transducer was calibrated using a sine wave generator and oscilloscope and has a pass-band transfer-impedance of $Z_T = -0.9$ dBΩ. The result in the 50 Hz to 200 kHz band is shown in Figure 2.8.

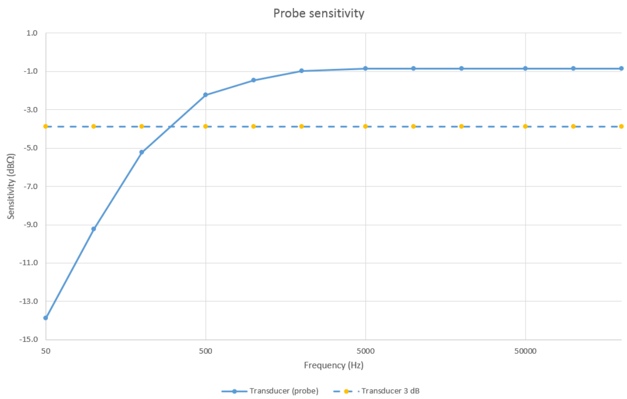


Fig. 2.8: Transfer-impedance of new current transducer (blue solid line)

2.5 The importance of the termination resistor

The termination resistor R_{MEAS} is a very important prerequisite. It not only serves to determine the frequency behavior of the current transducer. It actually has three functions:

1. R_{MEAS} determines the onset and level of the “flat” part of the frequency response of the transducer at the right hand side of the cut-off frequency ω_{CO} in Fig. 2.6;

2. R_{MEAS} limits the output voltage level caused by transients in the measured current. Due to Faraday's Law, the substitute voltage source driving the behavior of the current transducer is equal to $V_2(t) = M_{12}(t) \cdot \frac{\mathrm{d}I_1(t)}{\mathrm{d}t}$ as shown in Equation 2.4. This driving voltage can become very large when dt is short! It could destroy the measuring instrument's input circuitry;

3. R_{MEAS} limits the peak flux developed in the metal or ferrite core (if any) of the transducer by drawing a load current which creates a flux in the core in the opposite direction compared to the original flux resulting from the mutual induction term. This behavior is described by Lenz' law, [1, p. 659]. That way a termination resistor helps to prevent saturation of the transducers' core. This opposing flux is also represented by the minus sign in Faraday's law (Equation 2.4).

2.6 A small magnetic field transducer

The "single turn" magnetic field (loop-)transducer, shown in Fig. 2.9, is also based on Faraday's Law. As with the current transducer described earlier, it operates on mutual induction. But, assuming the flux producing current is hidden somewhere in the environment, a mutual-induction model to find the corresponding flux that drives the transducer is no longer appropriate. It is easier to start with the flux-density, B, that needs to be measured and multiply it with the effective area of the measuring loop-transducer, A_E of the transducer to find the total flux, Φ, in the transducer loop. Apart from this, the equations are very similar to those seen for the current transducer: in Equation 2.3 the term

$M_{12} \cdot I_1(t)$ is replaced by $B(t) \cdot A_E$. Possibly multiplied by the number of turns, N, for a multi-turn coil because $\Phi(t)$ has its inductive effect on each separate turn of the coil. $B(t)$ is the flux density within the effective area of the loop-transducer (A_E)[2].

As for the electric field probes in Chapter 1, the probe diameter or size is supposed to be smaller than a fraction of the shortest wavelength present in the fields to be measured for the equations in this document to be valid: the low-frequency approximation.

Equation 2.5 for the transducer voltage in the frequency domain changes into Equation 2.11 for the single–loop version of such a transducer and Equation 2.12 for a multiturn coil type transducer. Obviously, the magnetic flux density $B(\omega)$ needs to be used here instead of $B(t)$.

$$V_2(j\omega) = -j\omega\Phi(\omega) = -j\omega B(\omega) \cdot A_E \quad \text{V} \quad \text{for a single-turn loop} \tag{2.11}$$

$$V_2(j\omega) = -j\omega\Phi(\omega) = -j\omega N \cdot B(\omega) \cdot A_E \quad \text{V} \quad \text{for an } N\text{-turn coil} \tag{2.12}$$

The magnetic flux density, B, is related to the magnetic field, H, by the relation in Equation 2.13. The dimension of the magnetic field H is Ampere/meter (A/m). B is expressed in Tesla (T) in the international system of units (SI).

$$B(j\omega) = \mu \cdot H(j\omega) \qquad \text{Wb/m}^2 \quad \text{or} \quad \text{T} \tag{2.13}$$

[2] The assumption that the flux density is constant over the entire effective area, A_E, is only valid if the source of the magnetic field is "far" away. Especially single turn loop probes are often used very near high-frequency equipment. Generally as a, so-called, "sniffer"–probe to see if there is any field present at some location. If anything needs to be said about magnitude of the actual field strength i.e. if a real *measurement* is attempted, the magnetic field officially needs to be integrated over the area of the loop to obtain an "average" value for the flux density B.

If the field is measured in an air environment (or vacuum) the permeability of vacuum is used: $\mu = \mu_0 = 4\pi \cdot 10^{-7}$ H/m. In theory this value needs to be multiplied with the relative permeability, μ_r of the actual environment. The product $B(\omega) \cdot A_E = \Phi(\omega)$ has the dimension Wb or Weber.

The most basic form of a magnetic field probe is a single turn loop which should in principle be shielded against electric fields. This results in a probe that looks like Fig. 2.9. Note that if a coax cable is used to achieve E-field shielding for the loop, there has to be an opening in this shield somewhere in this loop (air-gap). Otherwise, the magnetic field will also be shielded!

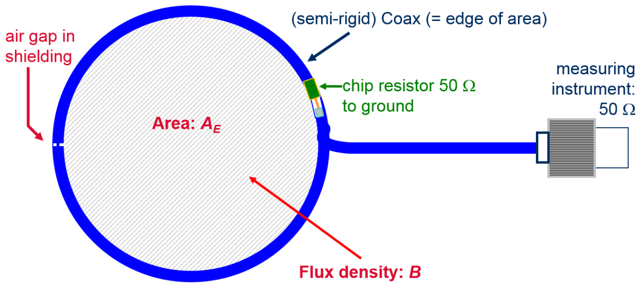


Fig. 2.9: A one-turn magnetic field transducer

The physical probe is shown in Fig. 2.10. To avoid short circuits the loop should preferably be insulated using e.g. heat shrinkable tubing before measurements inside live equipment are attempted.

Fig. 2.10: A physical magnetic field pick-up probe

2.7 On the label “B-dot probe”

The small loop probe, shown in Fig. 2.10, is called a “Bdot” probe e.g. in the MIL-STD-461 [2, page 152]. This is because the measuring principle is based on the derivative of the flux–density with respect to time: $\dot{B} = \frac{\mathrm{d}B}{\mathrm{d}t} = \frac{\mathrm{d}\mu_0 \cdot H}{\mathrm{d}t}$.

2.8 A larger coil type magnetic field transducer

For lower frequencies, a larger area can be used with more turns of wire around it, a real coil. A specimen is shown in Fig. 2.11.

The coil in Fig. 2.11 has an aluminum strip around it which also holds the connector. Apart from a mechanical support for the connector it doubles as an electric-field screen. As with the single loop probe, there needs to be an air-gap in this screen to “open it up to magnetic fields”. This air-gap is shown in Fig. 2.12.

Fig. 2.11: Magnetic field pick-up coil for lower frequencies

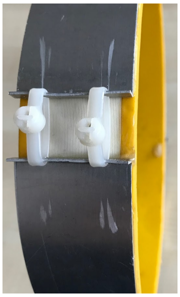


Fig. 2.12: Air gap in the shield of the magnetic field pick-up coil

2.9 Characteristics of the multiturn coil transducer

Equation 2.7 can be used for loop and coil probes too for calculation of the probe cut-off frequency. The induction L_{probe} can be measured using and LCR–bridge. R_{MEAS} is the termination resistor used on the measuring instrument.

The coil transducer of Fig. 2.11 has the following properties:

1. Diameter: $D_{COIL} = 0.2$ m;
2. Area: $A_E = 3.14 \cdot 10^{-2}$ m^2
3. Turns: $N = 58$ - wire-wrap size wire (AWG 30);
4. Inductance: $L_{COIL} = 925$ μH;
5. Cut-off frequency @ $R_{MEAS} = 50\ \Omega$: 8.6 kHz;

In line with Equation 2.10 we now use Equation 2.12 for the *substitute-voltage source*. The (asymptotical) output voltage response below the cut-off frequency, ω_{CO}, divided by the magnetic flux density rises proportional to frequency. This is represented by Equation 2.14

$$\frac{V_{out}(j\omega)}{B(j\omega)} = j\omega\,[N \cdot A_E] \quad \mathrm{V/T} \quad \omega << \omega_{CO} \tag{2.14}$$

the voltage output above the cut-off frequency divided by the magnetic flux density to be measured, the transducers *calibration level*, is given by Equation 2.15.

$$\frac{V_{out}(j\omega)}{B(j\omega)} = \frac{N \cdot A_E \cdot R_{MEAS}}{L_{COIL}} \quad \mathrm{V/T} \quad \omega >> \omega_{CO} \tag{2.15}$$

A graph of the –absolute value of– this transfer-function looks like Fig. 2.6 (replace I_1 by B).

In specifications of coil or loop transducers it is customary to provide a graph of the number of dBμV that have to be added to the measured output in order to find the correct field level. This correction is also called the Antenna-factor (AF) of the transducer. The antenna factor is defined as Equation 2.16.

$$AF_{\text{COIL}} = \left|\frac{B(j\omega)}{V_{out}(j\omega)}\right| \;\text{T/V} \quad \left(\text{i.e.} \quad |V_{out}(j\omega)| = \frac{|B(j\omega)|}{AF_{\text{COIL}}}\right) \;\text{V} \tag{2.16}$$

The components of this antenna factor below and above the cut-off frequency are calculated by Equations 2.17 and 2.18.

$$AF_{LF} = \left|\frac{1}{j\omega N \cdot A_E}\right| \quad \text{T/V} \qquad \omega << \omega_{CO} \tag{2.17}$$

$$AF_{HF} = \left|\frac{L_{COIL}}{N \cdot A_E \cdot R_{MEAS}}\right| \quad \text{T/V} \quad \omega >> \omega_{CO} \tag{2.18}$$

(hence $AF_{HF} = 10^{-5}$ T/V for the transducer in Fig. 2.11)

Fig. 2.13 shows the graph of these antenna factor asymptotes.

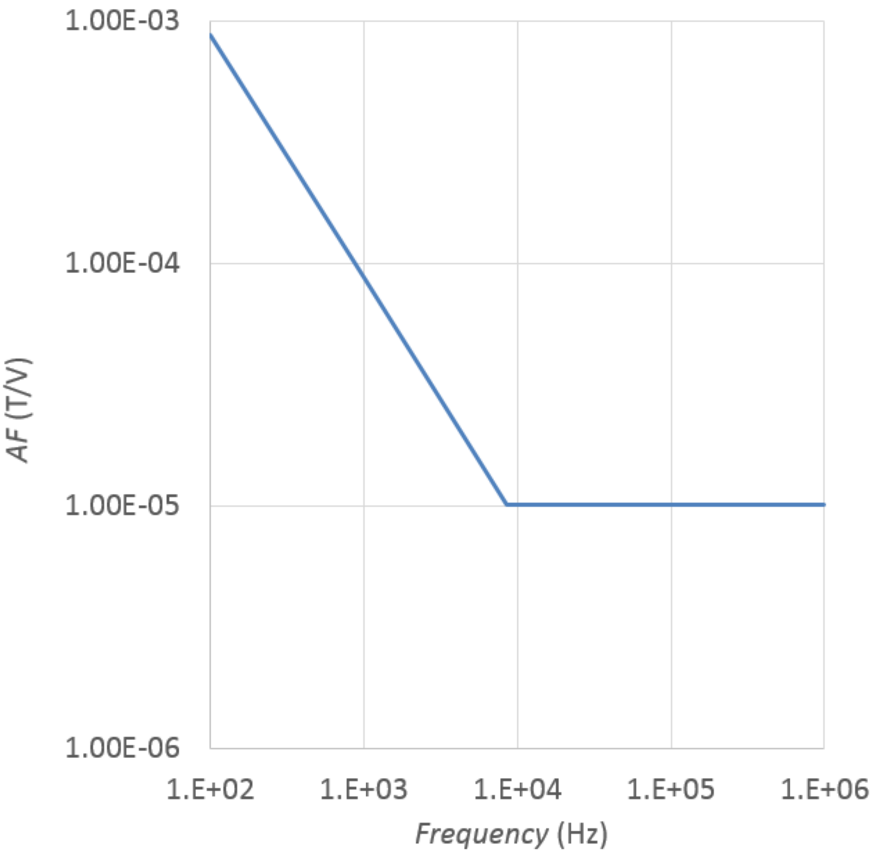


Fig. 2.13: Antenna Factor of the coil transducer of Fig. 2.11

2.10 Tuning the usable frequency range of magnetic probes

The cut-off frequency of capacitive probes, especially in the low frequency ranges, can be tuned relatively easy by increasing either the load or termination resistor or the parallel capacitance. See Section 1.6 on page 13. Modifying the termination resistor in a coil-type transducer to lower the usable frequency range is a theoretical option: a *smaller* resistor is required as is obvious from Equation 2.8 on page 20. To lower the cut-off frequency of the current transducer in Fig. 2.7 to 50 Hz a termination resistor of $R_{MEAS} = 8.5\ \Omega$ is required. Maybe an option for an active probe using an amplifier.

The other –passive– option, in line with the capacitive probe example, is to increase the inductance of the measuring coil. For e.g. the coil transducer, to use another example, of Fig. 2.11 that would require a coil inductance of $L_{COIL} = 160$ mH with a 50Ω termination. Instead of $N = 58$ the number of turns would go up to $N = 763$. The antenna factor would go down to $AF_{HF} = 1.34 \cdot 10^{-4}$ so it would make the transducer more sensitive in addition to lowering its cut-off frequency.

Placing an external inductor, L_{EXT}, *in series* with a magnetic transducer to be able to retain the $R_{MEAS} = 50\Omega$ and achieve an $F_{CO} = 50$ Hz or lower is probably not a good idea as it will reduce the sensitivity[3] of the transducer and increase the risk of parasitic ringing.

A good compromise is to terminate a magnetic transducer with the regular $R_{MEAS} = 50\Omega$ and use a lag-lead type, passive, filter to extend the frequency range downward. This is demonstrated on the example of the current transducer of Fig. 2.7 on page 21. To do the calculations, a model is made in LTSPICE.

[3] The calculations are left as an exercise for the interested reader.

2.11 Simulating the behavior of a magnetic transducer

LTSPICE is available from Analog Devices at https://www.analog.com/en/design-center/design-tools-and-calculators/ltspice-simulator.html#. The model for the current probe of Fig. 2.7 is shown in Fig. 2.14. The mutual inductance is modeled by using a transformer as input, driven with a *current* source of $I1 = 1$ A. An LTSPICE transformer is built using two separate inductors, $L1$ and $L2$, and specifying a coupling factor $K1$ between them.

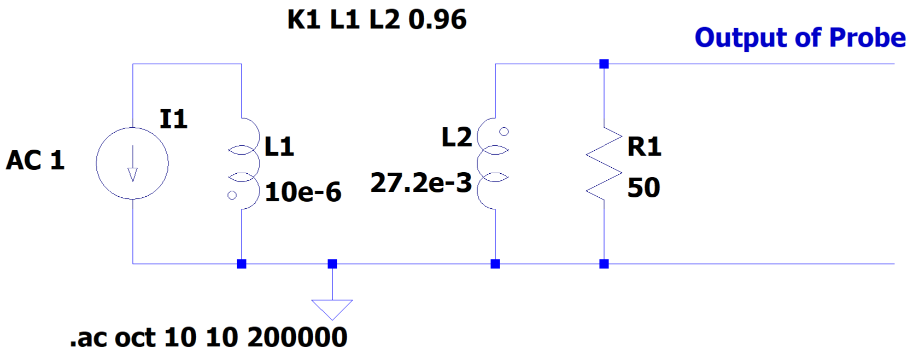


Fig. 2.14: The LTSPICE diagram for the transducer of Fig. 2.7

Running LTSPICE, the frequency response shown in Fig. 2.15 appears. It was made to match Fig. 2.8 on page 22 by tweaking[4] the values of $L1$ and $K1$.

Fig. 2.16 shows the modified LTSPICE model with the correction network.

Fig. 2.17 shows both model response outputs:

- The "Output of Probe" in blue
- The "Extended LF range output" in red

[4] Tune simulation models to match actual measurements.

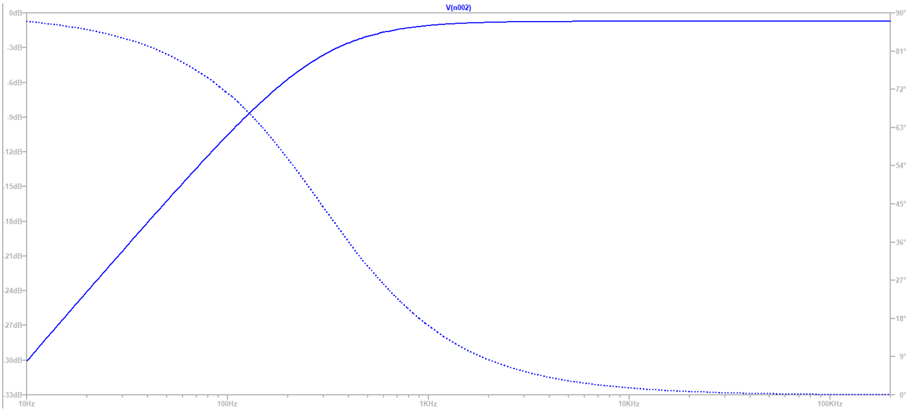


Fig. 2.15: Simulated response for the transducer of Fig. 2.7

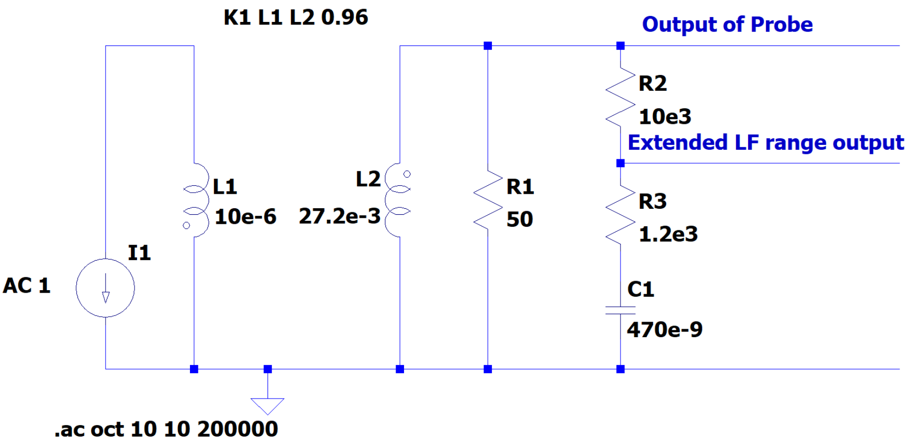


Fig. 2.16: Model for transducer of Fig. 2.7 with correction network

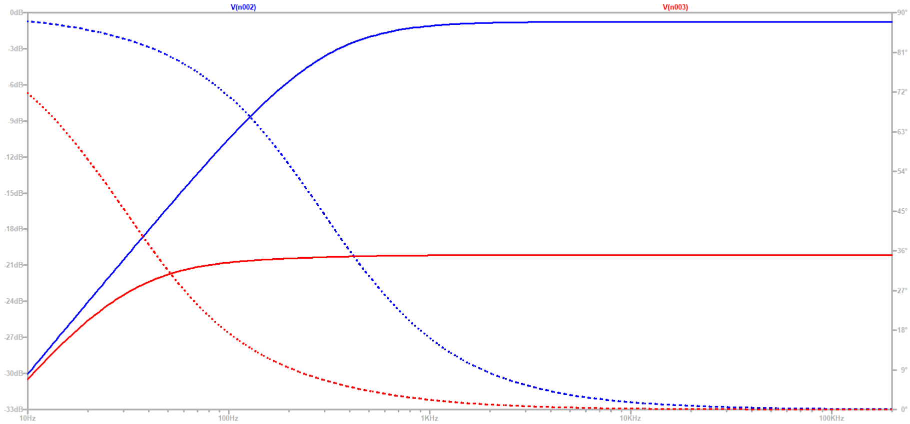


Fig. 2.17: Simulated response for the transducer of Fig. 2.7 without (blue) and with (red) correction

The extended frequency range comes with a considerable reduction in sensitivity as Fig. 2.17 clearly shows. Around 20 dB in this case. This could be acceptable if the current to be measured is large enough.

2.12 Passive probes show *phase shift*

As shown in Fig. 2.15 and Fig. 2.17, probes not only manifest an amplitude-response but also a *phase-response* with frequency. This is relevant if the measured currents are used together with the measured voltages e.g. to calculate the instantaneous power at some moment in time. The same problem occurs when e.g. the 50 Hz mains voltage is displayed together with the measured current in the time domain. This same phenomenon is observed in capacitive probes as described in Chapter 1.

2.13 Characteristics of the single turn Bdot-probe

The Bdot-probe shown in Fig. 2.10 can be analyzed in the same way as the multiturn coil probe. Measuring the loop inductance may be a challenge. Since the probe shown has a built in $50\ \Omega$ resistor, the LCR-bridge will measure that resistance. Since it is not necessary to calibrate these pick-up loops accurately because of their use for relative experiments, an approximation can be made using e.g. a web-based utility as https://www.eeweb.com/tools/loop-inductance. We find:

1. Diameter: $D_{LOOP} = 0.029$ m (of loop center conductor);
2. Area: $A_E = 66 \cdot 10^{-5}$ m^2
3. Turns: $N = 1$ -;
4. Inductance (calculated): $L_{LOOP} = 63$ nH;
5. Cut-off frequency @ $100\ \Omega$[5]: 253 MHz;

[5] $100\ \Omega$ is used since a $50\ \Omega$ resistor is already built in. It serves to prevent reflections on the transmission line –semi-rigid 50Ω coax– from which the probe is built between the air gap and this termination. The resistor should be added to the $50\ \Omega$ impedance of the measuring instrument.

$$AF_{LF} = \left| \frac{1}{j\omega A_E} \right| \quad \mathrm{T/V} \qquad \omega << \omega_{CO} \tag{2.19}$$

$$AF_{HF} = \left| \frac{L_{LOOP}}{A_E \cdot R_{MEAS}} \right| \quad \mathrm{T/V} \quad \omega >> \omega_{CO} \tag{2.20}$$

$$(AF_{HF} = 9.5 \cdot 10^{-7}\ \mathrm{T/V} \text{ for the transducer in Fig. 2.10}) \tag{2.21}$$

Fig. 2.18 shows the graph of the antenna factor asymptotes for the loop-probe.

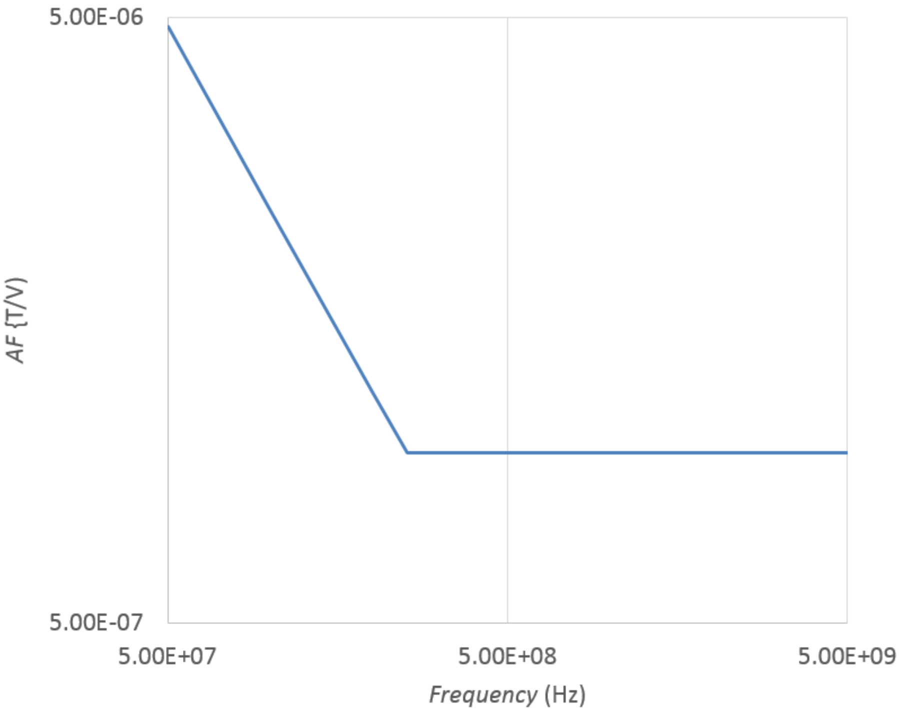


Fig. 2.18: Antenna Factor of the single loop transducer of Fig. 2.10

A

Comparing EM measurements around the globe

A.1 Building transducers is very rewarding

Having a set of electromagnetic (EM)-transducers is indispensable to obtain a basic knowledge of the world of electromagnetic interference (EMI) and setting your first steps towards building electromagnetic compatibility (EMC)-electronic designs.

You can buy a set on the market. But building your own set of EM transducers is a more intimate way to teach yourself how EM-fields work. That was what inspired me to write this book.

If knowledge of the behavior of fields is your goal, build your own transducers and experiment with various shapes and sizes!

A.2 Prove EMC by compliance to standards

On the professional level, you will probably have to convince your peers that the equipment you built, complies to the limit values specified in the EMC standards for the environment your equipment is intended to work in.

In that situation your “home made probes” may be less useful. You need to work with calibrated equipment.

You could calibrate your manufactured device e.g. as shown in Section 2.4 in Fig. 2.8 on page 22.

But of course, the curve in Fig. 2.8 is only part of the behavior of your current transducer. How useful is it at frequencies below your cut-off frequency? Where will we find the first resonance in the high frequencies?

For a current transducer as shown in Fig. 2.7 the advantage is the closed magnetic ferrite ring will make sure the measured induction effects are only a result of the currents in the wire(s) passing through the center hole. But what is the behavior of this ferrite ring over frequency?

But for the ring shaped B-dot probe shown in Fig. 2.9 on page 25, calibration is , in essence, not possible.

The same goes for our D-dot probe of Fig. 2.10 on page 25.

This means “home built measurement equipment” is excellent for educational purposes:

1. They are cheap;
2. Nothing comes close in increasing your EM-insights.

Home built probes are also fine for ***relative*** engineering tests, where you wish to find out whether you have any fields and what the effect is of measures you take: 10 dB better (or worse!)? Or rather 40 to 60 dB improvement?

A.3 Schuuring's Law

One of the former managers of the EMC-lab of the Hollandse Signaal Apparaten[1] B.V. in Hengelo, the Netherlands, D. Schuuring M.Sc. had the following philosophy:

"It is not so important that the prescribed measurement procedure in an EMC-standard is strictly correct. It is important that this same test will give identical results wherever on the globe you perform this measurement"

At least locally in Hengelo, this is known as the Schuuring Law.

The essential requirement for any compliance testing is that the results will be equal, regardless of where you perform the tests.

[1] Currently Thales Nederland B.V.

A.4 Build a standard B-dot probe

Even though any home building will lead to the label “hobby-device”, building the B-dot probe advertised in the MIL-STD-461G, [2, p.128], will at least give you the opportunity to compare your findings with those of your peers abroad.

A.4.1 The RS101 loop sensor

The specification for the 4 cm loop sensor of the RS101 test in [2] is shown in Fig. A.1a. The home-built version is shown in Fig. A.1b.

(1)	Diameter:	4 cm
(2)	Number of turns:	51
(3)	Wire:	7-41 Litz wire (7 strand no. 41 AWG)
(4)	Shielding:	Electrostatic
(5)	Correction Factor:	See manufacturers data sheet (i.e. your responsibility)

(a) Loop sensor specification

(b) Home-built RS101 loop sensor

Fig. A.1: A home-built RS101 loop sensor with “built-in” 5 cm distance

A.4.2 Build the RS101 loop sensor

The loop sensor shown in Fig. A.1b is built using 64 mm long section of 40 mm diameter poly vinyl chloride (PVC) –plastic– plumbing tube available at hardware stores. The wall thickness should be 3 mm.

In the loop sensor design of Fig. A.1b a Bayonet-Neill-Concelman (BNC) connector is mounted in a plastic 40 mm tube lid also available at your hardware store. This lid covers the 2.6 mm wide, 1.5 mm deep slot holding the coil. The slot is made using a lathe.

A 1 mm hole is drilled in the edge of the slot to allow the coil wires to reach the BNC-connector.

The relevant dimensions are shown in Fig. A.2.

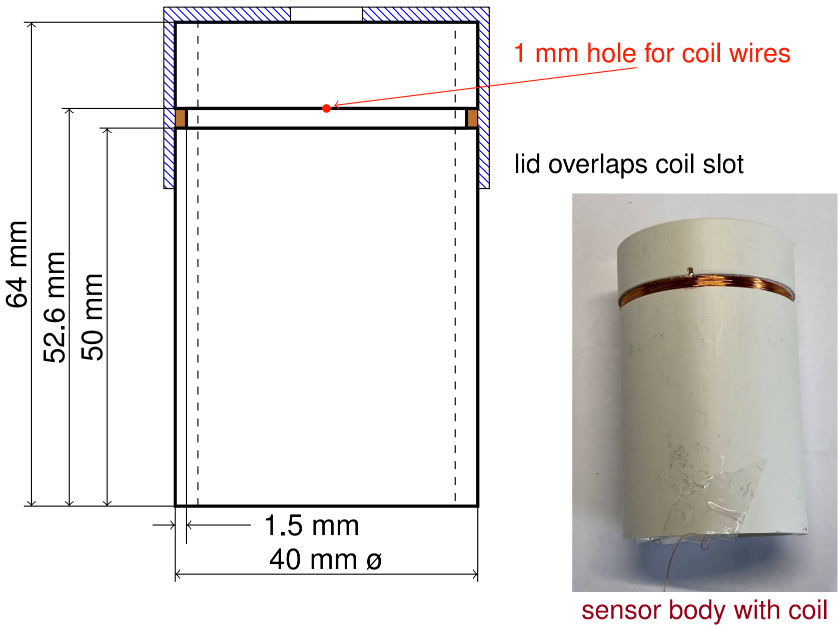


Fig. A.2: dimensions of the loop sensor coil body

The coil is wound using 0.15 mm enameled copper wire. 51 turns, as specified. Beginning and end of the wire are fed into the 1 mm hole at the edge of the coil slot.

These wire ends should be long enough to be able to solder them to the BNC-connector while the lid is still detached from the coil body. 60 mm should be O.K.

Before soldering the wires these should be provided with 50mm thin plastic isolation tubes –taken from other insulated wires–. Shove them carefully over the wires up to the 1 mm hole and secure them there with hot glue. Keep the two wires together using yarn or thin lacing tape.

Then solder them to the BNC connector terminals. Secure the wires with hot glue to the connector body as strain relief for these thin wires.

After testing the coil is working[2], the tube lid can be glued to the coil body (using hard-PVC-glue).

Note 1: The coil body has 50 mm of plastic tube between the coil slot and the "measuring end" of the device. Officially, the distance between the radiator coil center (see Section A.5) and the center of the loop sensor. Refer to https://www.solar-emc.com/9230-1.html, Fig.1 for details. Depending on the way the coil is used, the current 50 mm PVC-pipe can be shortened using sand-paper or a sander.

Note 2: The specification (item (4) in Fig. A.1a) requires *electrostatic* shielding. This can be achieved by wrapping a strip of sticky copper tape over the coil on the outside of the tube lid. DO NOT FORGET the mandatory "air-gap" as shown e.g. in Fig. 2.12.

Once you decided to go for electrostatic shielding, this copper tape needs to connect to the outside body of the BNC connector.

Place a strip of sticky copper tape from the shield into the hole for the BNC before mounting the connector. This only works if you use the connector shown in Fig. A.3b. Verify good contact using a (milli) ohmmeter.

[2] You could measure its inductance: the coil used during the hands-on experiment afternoon measures $L_{\text{loop-sensor}} \approx 225\ \mu\text{H} + 7\ \Omega$. Or you could build the radiator first and then try the calibration.

(a) Unshielded: *Insulated* BNC O.K. **(b)** Shielded: use *Grounded* BNC

Fig. A.3: Available BNC connectors

If you decide NOT to SHIELD[3] electrostatically, either connector in Fig: A.3 can be used. The insulated version, Fig. A.3a, is easier for connecting the two thin wires.

[3] The professional version in Fig.4 at https://www.solar-emc.com/9230-1.html does not appear to be shielded electrostatically i.e. you would not be the only one.

A.5 Build a standard radiator coil

A weak spot in the specification of Fig. A.1a is point (5).

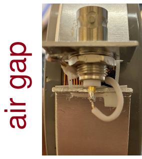
But the MIL-STD-461G RS101, [2, page 128] also provides you with the design specification of a 12 cm radiating loop to experiment and calibrate with.

A.5.1 Build the RS101 radiator coil

This specification of [2] is copied into Fig. A.4a. A completed model for use in experiments is shown in Fig. A.4b.

(1)	Diameter:	12 cm
(2)	Turns:	20
(3)	Wire:	12 AWG = 2 mm diameter insulated copper
(4)	Shielding:	not required
(5)	Flux density	9.5×10^7 pT/A of applied current at 5 cm from the plane of the loop

(a) Radiator specification



(b) Home-built RS101 radiator

Fig. A.4: A home-built RS101 radiator

A length of 12 cm diameter PVC pipe can be used to wind the coil of Fig. A.4. The coil shown is wound on a lid for a 110 mm PVC pipe. This way the closed end of the pipe can serve as a support for the loop sensor coil described in Section A.4.1.

The radiating coil has a 20 turn single-layer of 0.81 mm (20 AWG) enameled copper wire in order not to make the device too heavy/bulky for experiments.

The radiator is built in the same way as the large coil described in Section 2.8 on page 26. This means that the actual coil is (almost –air gap–) enclosed with an aluminum strip, holding the connector and keeping the wires in place.

Although the RS101 radiator coil does not require electrostatic shielding, The aluminum strip can serve as one. The coil could be used as a sensor also to measure fields.

As we will not be using the radiator for high currents –no suitable generator available in the hands-on set– a BNC connector is used.
Of course the model shown in Fig. A.3b is used to actually obtain the electrostatic shielding.

For completeness the air gap construction is shown to the left of the yellow arrow in Fig. A.4.

A.5.2 Accessing the expected radiator field output

We like to create a large field strength locally with our radiator coil.

This field strength is proportional to current, I, and the number of turns, N, in the coil.

To find the –vertical component– of the field produced by out radiator coil, we must add the contributions of each individual wire turn.

Fig. A.5 shows the vertical component of the field strength, B_y, of one single turn with radius R.

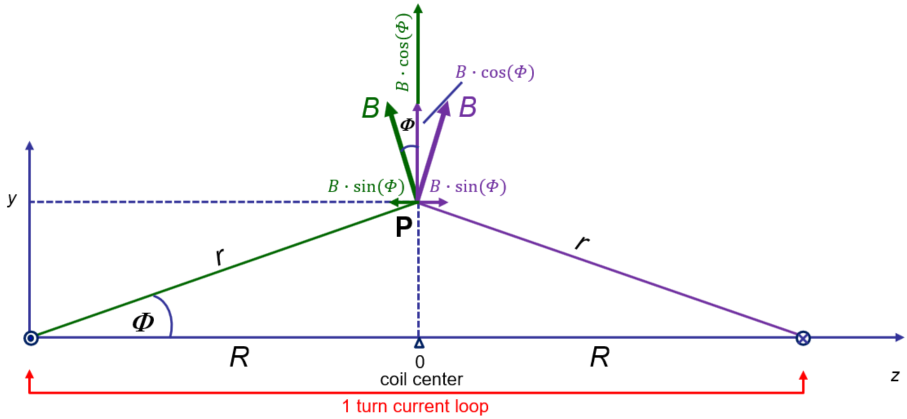


Fig. A.5: Field of one circular current-loop

The accumulated vertical component of the magnetic flux density, B_y, at distance y above the center of the loop[4] is given in Equation A.1.

$$B_y = \frac{\mu_0 I}{2} \cdot \frac{R^2}{(y^2 + R^2)^{3/2}} \quad \mathrm{T} \tag{A.1}$$

Where I is the current in the loop, y the height above the center of the circular loop, in the same dimension as the loop radius, R.

The parameter $\mu_0 = 4\pi 10^{-7}$ H/m is the permeability of vacuum. We will use this value for our radiator coil in air.
If the radiator coil is used in any other medium, of course, the permeability of that medium should be used.

Equation A.1 turns into Equation A.2 for $y = 0$, giving the perpendicular value of B at the center of our single turn.

$$B_0 = \frac{\mu_0 I}{2R} \quad \mathrm{T} \tag{A.2}$$

[4] The complete derivation can be found in Chapter 12 Section 5 at https://phys.libretexts.org/Bookshelves/University_Physics/Book%3A_University_Physics_(OpenStax)/Book%3A_University_Physics_II_-_Thermodynamics_Electricity_and_Magnetism_(OpenStax).

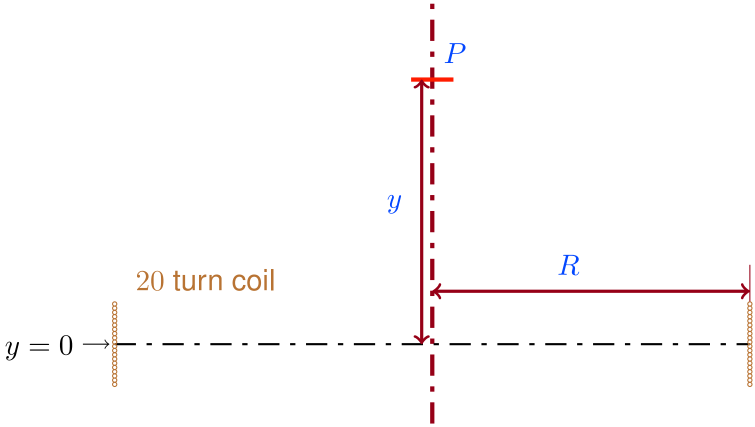


Fig. A.6: Configuration of the RS101 radiator

A.5.3 Create an Excel sheet to calculate the field at *y*

	A	B	C	D	E
1			N	20	Turns
2			μ_0	1.25664E-06	H/m
3	y		I	0.003	A
4	0.050	m	R	0.058	m
5			wire diameter	0.0008	m
6			Depth turn: d (m)	$B_{(y+d)}$ (T)	
7		-10	-0.0076	1.70986E-08	
8		-9	-0.0068	1.67639E-08	
9		-8	-0.006	1.6434E-08	
10		-7	-0.0052	1.6109E-08	
11		-6	-0.0044	1.57891E-08	
12		-5	-0.0036	1.54742E-08	
13		-4	-0.0028	1.51644E-08	
14		-3	-0.002	1.48598E-08	
15		-2	-0.0012	1.45604E-08	
16	loop plane	-1	-0.0004	1.42662E-08	
17		1	0.0004	1.39772E-08	
18		2	0.0012	1.36935E-08	
19		3	0.002	1.34149E-08	
20		4	0.0028	1.31416E-08	
21		5	0.0036	1.28735E-08	
22		6	0.0044	1.26106E-08	
23		7	0.0052	1.23528E-08	
24		8	0.006	1.21E-08	
25		9	0.0068	1.18524E-08	
26		10	0.0076	1.16097E-08	
27			B_Y	2.841E-07	T

Fig. A.7: Calculation total field

In cell A4 of the Excel sheet in Fig. A.7 we place the height, y, above the center of the radiator coil.

Rows 7 through 26 contain the data for each turn in the coil.

Column B shows an offset -10 through 10 to indicate rows above and below the “0 mm” line, exactly between turn -1 and 1.

The y value in the calculations is corrected for each turn by the distance from the center of each turn and the “0 mm” line.

This y offset is shown in column C, C7 through C26.

Column D contains the flux density contributions of each turn of wire using Equation A.1.

Let us take cell D15 as an example. Equation A.1 is written, for cell D15, as Equation A.3.

$$D15 = \$D\$2 * \$D\$3 * \$D\$4 \wedge 2/2/((\$A\$4 + C15) \wedge 2 + \$D\$4 \wedge 2) \wedge (3/2) \tag{A.3}$$

This equation is copied into all cells (D7 through D26).

Finally, the total flux density in point y is calculated as
$D27 = SUM(D7 : D26) = 2.841E - 7$ T(esla).
This is equal to $B_Y = 284100$ pT
We can also calculate $20\log(B_Y) = 109$ dBpT.

If we divide this number by the coil current in cell D3, 3 mA, we obtain item (5) in the radiator specification of Fig. A.4a on page 44:
$B_{5\ cm} = 9.5 \times 10^7$ pT/A.

Using the Excel sheet, it can be checked that this number, the pT/A, does not change if a different current is inserted i.e. is only dependent on the geometry of the coil and the measurement point, y.

A.5.4 Calibrate your RS101 loop sensor

Determine the Antenna Factor of the sensor

The RS101 loop sensor can be analyzed in the same way as the multiturn coil transducer in Section 2.9. After completion, the device's coil measured $L_{sensor} = 225\ \mu$H in series with $R_{ESR} = 7\ \Omega$.

The induced voltage, $V_{INDUCED}$, in the RS101 sensor is loaded with the coil's inductance, L_{SENSOR}, in series with the coil's equivalent series resistor (ESR), R_{ESR}, and the termination resistor, R_{MEAS}, of the measuring instrument.

The induced voltage is calculated as Equation A.4.

$$V_{INDUCED}(j\omega) = j\omega_{SOURCE}(B_{SOURCE} \cdot A_{1TURN} \cdot N) \qquad \text{V} \qquad \text{(A.4)}$$

Where:

ω_{SOURCE}	The radian frequency of the magnetic field source
B_{SOURCE}	The flux density of the magnetic field in the coil
A_{1TURN}	The area of 1 turn of the coil wire
N	The number of turns in the coil

It is important to remember the term $B_{SOURCE} \cdot A_{1TURN} \cdot N$ in Equation A.4 is a *constant* in our case here and is the equivalent of M_{12} in Equation 2.9 and 2.10 on page 20.

B_{SOURCE} only depends on the current –constant over frequency– in the RS101 radiator coil and the distance –fixed–between the centers of the radiator and the loop sensor coils when aligned as indicated in the MIL-STD-461G RS101 description, [2, page 134], partly copied in Fig. A.8.

The equivalent circuit for the RS101 loop sensor can be drawn as Fig. A.9.

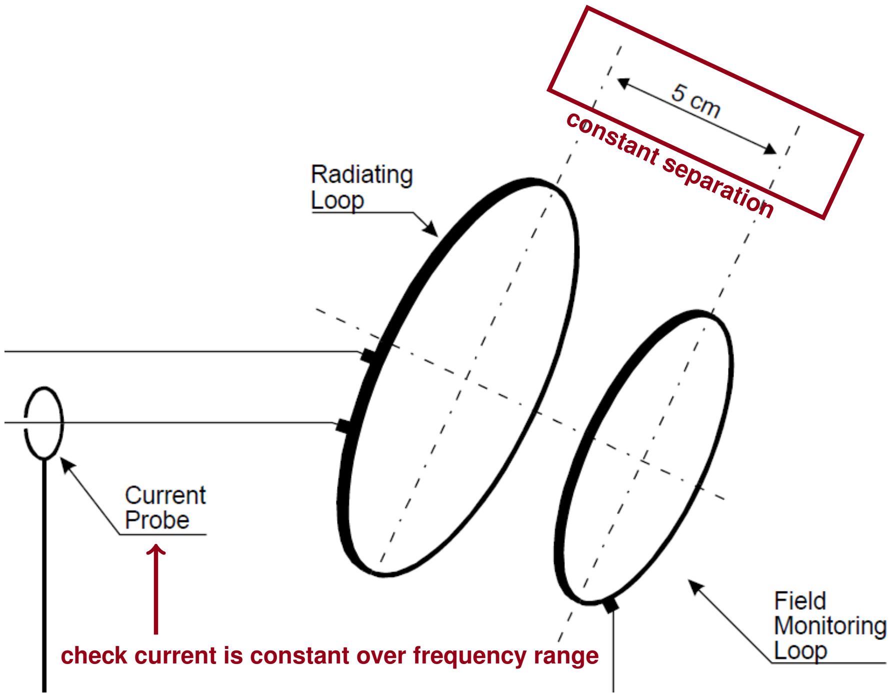


Fig. A.8: Alignment RS101 radiator and loop sensor coils

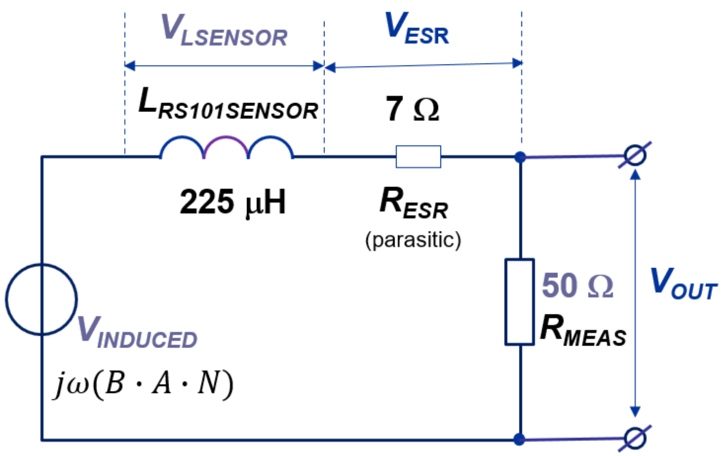


Fig. A.9: Equivalent circuit for the RS101 loop sensor

As the R_{ESR} is non-trivial here, it should be included in the analysis. Its effect is that the cut-off frequency is slightly higher now. See Equation A.5.

$$\begin{aligned} \omega_{CO} =& \frac{R_{ESR} + R_{MEAS}}{L_{SENSOR}} && = 253 \times 10^3 && \text{rad/s} \\ F_{CO} =& \frac{\omega_{CO}}{2\pi} && = 40.3 \times 10^3 && \text{Hz} \end{aligned} \tag{A.5}$$

compared to **without** R_{ESR}:

$$F_{CO} = \quad = 35.4 \times 10^3 \quad \text{Hz}$$

The second effect is that it *attenuates* the output voltage i.e. acts as a voltage divider with an attenuation shown in Equation A.6.

$$Atten_{sensor} = \frac{R_{MEAS}}{R_{MEAS} + R_{ESR}} = 0.88 \quad - \tag{A.6}$$

The Antenna-Factor of the home-built RS101 loop sensor then becomes:

$$\begin{aligned} AF_{\text{sensor}} &= \left| \frac{B}{V_{out}}(j\omega) \right| \\ AF_{\text{sensor}}(\omega << \omega_{CO}) &= \left| \frac{1}{j\omega(Atten_{sensor} \cdot N \cdot A_{1TURN})} \right| \quad \text{T/V} \\ AF_{\text{sensor}}(\omega >> \omega_{CO}) &= \left| \frac{L_{SENSOR}}{Atten_{sensor} \cdot (R_{MEAS} + R_{ESR}) \cdot N \cdot A_{1TURN}} \right| \quad \text{T/V} \end{aligned} \tag{A.7}$$

$$\therefore AF_{\text{sensor}}(\omega >> \omega_{CO}) = \left| \frac{L_{SENSOR}}{R_{MEAS} \cdot N \cdot A_{1TURN}} \right| \quad \text{T/V} \tag{A.8}$$

Tuning based on the construction details of the sensor built

A slight deviation from the official specifications in [2, page 128] is the (average) diameter of the coil.
Since the coil body is made of 40 mm diameter PVC tube and the complete coil is *inside* this diameter, the actual number of turns needs to be

corrected to bring the term $N \cdot A_{1TURN}$ in line with the original specification.
The specified diameter of 40mm at 51 turns leads to $N \cdot A_{1TURN} = 51 \cdot \dfrac{\pi \cdot (40 \times 10^{-3})^2}{4} = 64 \times 10^{-3}\ \mathrm{m}^2$

The actual diameter of the manufactured coil for the hands-on experiments is 38 mm.
To obtain the correct "turns times area", the number of turns required is: $N' = \dfrac{64 \times 10^{-3}}{\dfrac{\pi \cdot (38 \times 10^{-3})^2}{4}} = 56$ which is the number of turns in the coil built. This implies the coil in Fig. A.1b should now behave as the specified RS101 loop sensor in Fig. A.1a.

Given the induction, $L_{SENSOR} = 225\ \mu\mathrm{H}$, $R_{MEAS} = 50\ \Omega$ and $N \cdot A_{1TURN} = 64 \times 10^{-3}\ \mathrm{m}^2$ we find $AF_{\mathrm{sensor}} = 70 \times 10^{-6}\ \mathrm{T/V} = 37\ \mathrm{dB}\frac{pT}{\mu V}$ for the high frequencies.

A graph of the Antenna Factor for the RS101 loop sensor, built for the experiments, is shown in Fig. A.10.

The challenge for the hands-on experiment session will be to calibrate the RS101 loop sensor using the RS101 radiator at 1000 Hz.

For those interested, the suggested procedure from the MIL-STD-461G, [2, page 129] is shown in Fig. A.11.

5.20.3.4 Procedures.
The test procedures shall be as follows:

a. Turn on the measurement equipment and allow sufficient time for stabilization.

b. Calibration.

(1) Set the signal source to a frequency of 1 kHz and adjust the output to provide a magnetic flux density of 110 dB above one picotesla as determined by the reading obtained on measurement receiver A and the relationship given in 5.20.3.2b(4).

(2) Measure the voltage output from the loop sensor using measurement receiver B.

(3) Verify that the output on measurement receiver B is within ±3 dB of the expected value based on the antenna factor and record this value.

Fig. A.11: Suggested calibration procedure in MIL-STD-461G

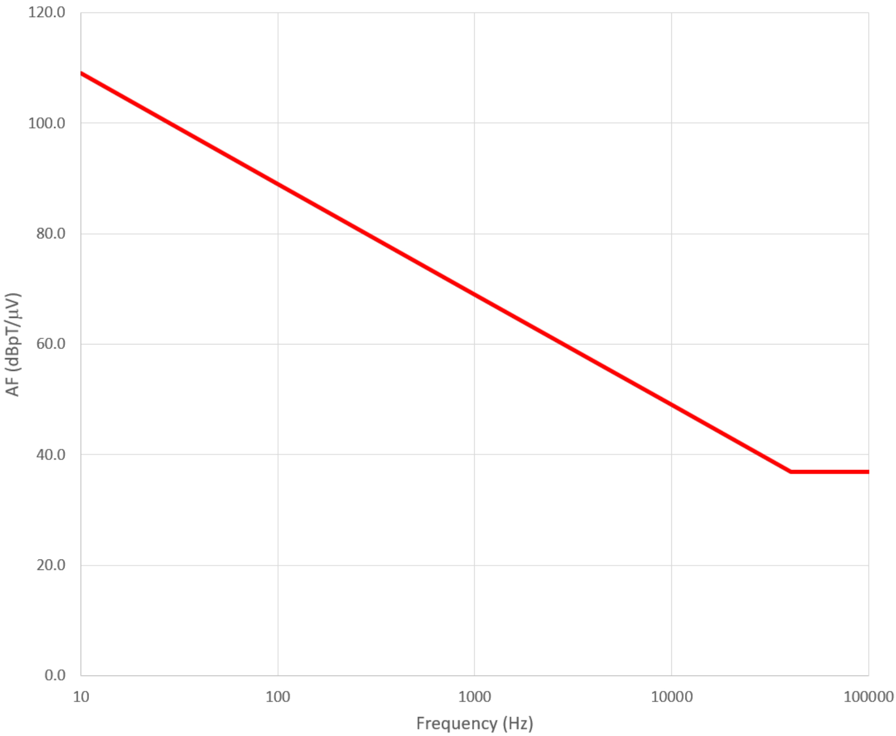


Fig. A.10: Antenna Factor for the RS101 loop sensor

More details can be found in the presentation handout (Homemade_Probes__Presentation_2023.pdf).

List of Abbreviations

AC	alternating current
AF	antenna factor
AWG	American wire gauge
BNC	Bayonet-Neill-Concelman
CM	Common-Mode
DC	direct current
ESR	equivalent series resistor
EEMCS	Electrical Engineering, Mathematics and Computer Science
EM	electromagnetic
EMC	electromagnetic compatibility
EMI	electromagnetic interference
GRP	ground reference plane
HSC	Historic Study Collection
PCB	printed circuit board
PVC	poly vinyl chloride
SI	international system of units
ESD	electrostatic discharge
GRP	ground reference plane
VLF	very low-frequency (3 to 30 kHz)

References

[1] R. Serway, C. Vuille, and J. Hughes, *College Physics*, 11th ed. Cengage learning, 2018. [Online]. Available: https://www.academia.edu/39954435/College_Physics_11th_Edition_Serway_Vuille.

[2] *MIL-STD-461G Requirements for the control of electromagnetic interference characteristics of subsystems and equipment* Dec. 2015. [Online]. Available: http://everyspec.com/MIL-STD/MIL-STD-0300-0499/MIL-STD-461_8678/.